ELSEVIER

European Journal of Mechanics B/Fluids 21 (2002) 371–380

EUROPEAN JOURNAL OF MECHANICS B/FLUIDS

A self-similar solution of a shock propagation in a dusty gas

Helfried Steiner *, Thomas Hirschler

Institute of Fluid Mechanics and Heat Transfer, Technical University Graz, Inffeldgasse 25, A-8010 Graz, Austria

Received 10 July 2001; received in revised form 24 October 2001; accepted 12 December 2001

Abstract

Analytic solutions are obtained for the unsteady, one-dimensional self-similar flow field between a strong shock and a moving piston behind it in a dusty gas. The dust is assumed to consist of small solid particles distributed continuously in a perfect gas. The dust's inertia and its solid phase behaviour strongly influence the wave propagation. It is shown that increasing the mass fraction of the dust does not automatically decelerate the shock front. Whether the shock is faster or slower than in the dust free case, rather depends on the volume fraction of the dust. This holds true whether the piston is accelerated, is moving at constant speed or is decelerated. © 2002 Éditions scientifiques et médicales Elsevier SAS. All rights reserved.

Keywords: Piston problem; Self-similarity; Dusty gas

1. Introduction

In most analytical approaches investigating the flow field behind a propagating shock in a dusty gas [1–4] the dust model which was thoroughly described by Pai [5] was used. This model assumes the dusty gas to be a mixture of small solid particles and a perfect gas. The dust phase comprises the total amount of solid particles which are continuously distributed in the perfect gas. On the one hand, the volumetric fraction of the dust lowers the compressibility of the mixture. On the other hand, the mass of the dust load may increase the total mass, and hence it may add to the inertia of the mixture. Both effects due to the addition of dust, the decrease of the mixture's compressibility and the increase of the mixture's inertia may markedly influence the wave propagation.

The present study investigates the impact of the two effects on the flow field between a shock and a moving piston behind it. In this particular configuration the flow field mainly depends on the interference of the sound waves emerging from the piston and the shock. Moreover, as this configuration allows for an analytical consideration, it serves as an appropriate test case to demonstrate the two aforementioned counterplaying effects of the dust.

Miura and Glass [6] obtained an analytical solution for a planar dusty gas flow with constant velocities of the shock and the piston moving behind it. As they assumed zero volumetric extension for the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the wave propagation.

For plane, cylindrical and spherical geometry Vishwakarma [7] computed an analytical solution for the flow field behind a strong shock propagating at non-constant velocity in a dusty gas. He assumed an exponential time dependence for the velocity of the shock, which allowed him to transform the conservation equations into a system of ordinary differential equations. The analytic solution he obtained is limited to one particular evolution of the shock velocity in time. Any particle path can be regarded as the path of an imaginary piston moving behind the shock. Thus, being part of the solution, the motion of any imaginary piston cannot be explicitly imposed as a boundary condition.

* Correspondence and reprints.
E-mail address: steiner@fluidmech.tu-graz.ac.at (H. Steiner).

PII: S0997-7546(02)01181-0

Dealing with analytic solutions for the present piston problem the self-similar solutions represent a very general approach. They allow for a shock which is propagated at constant as well as variable velocity in planar, cylindrical or spherical geometry. The path of the piston is imposed as a boundary condition. Thereby, an accelerated, a decelerated, or a constant-velocity piston can be specified. However, self-similarity requires the velocity of the shock and the velocity of the piston to be proportional to the same power law in time and the strong shock assumption. Under these two assumptions the conservation equations can be reduced to a system of ordinary differential equations, which do not explicitly depend on the time. In the dust free case this self-similarity solution was extensively discussed by Sedov [8]. It is the aim of this work to extend this classical self-similar approach to the dusty gas case.

2. Basic equations

The conservation equations for an unsteady, plane, cylindrically or spherically symmetric flow field (see Fig. 1) between a shock and a piston moving behind it in a mixture of a perfect gas with small solid particles [5] may be written as

$$\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \rho\left(\frac{\partial u}{\partial r}\right) + j\left(\frac{\rho u}{r}\right) = 0, \tag{1}$$

$$\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right] + \left(\frac{1}{\rho}\right)\left(\frac{\partial p}{\partial r}\right) = 0, \tag{2}$$

$$\left[\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r}\right] + \left(\frac{p}{\rho^2}\right)\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] = 0, \tag{3}$$

where ρ is the mixture's density, u the velocity, p the pressure, e the mixture's specific internal energy and r and t the space- and the time co-ordinate, respectively. The geometry factor j can be either 0, 1 or 2 respecting plane, cylindrical or spherical geometry.

In Fig. 2 the dusty gas is shown in its reference state. The solid particles are continously distributed in the perfect gas and in their totality shall be referred to as dust. The reference state is specified in terms of p_0, ρ_0 and T_0 and two parameters k_{dust} and G which account for the dust load. The mass of the mixture m is constituted by the mass of the dust m_{dust} and the mass of the perfect gas m_{gas}:

$$m = m_{\text{gas}} + m_{\text{dust}}.$$

The parameter k_{dust} defined as

$$k_{\text{dust}} := \frac{m_{\text{dust}}}{m} \tag{4}$$

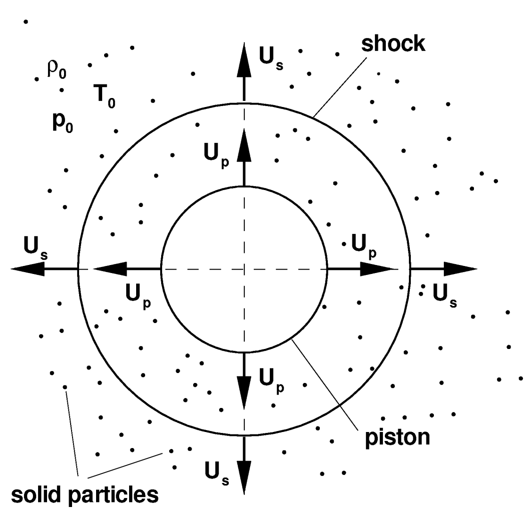

Fig. 1. Sketch of the flow configuration. The considered flow field is spherically symmetric and bounded by the piston and the shock, respectively. U_s is the velocity of the shock, U_p the velocity of the piston. The dusty gas is assumed to be a mixture of a perfect gas with small solid particles. p_0, T_0 and ρ_0 are the pressure, the temperature and the density of the mixture, respectively, at the reference state.

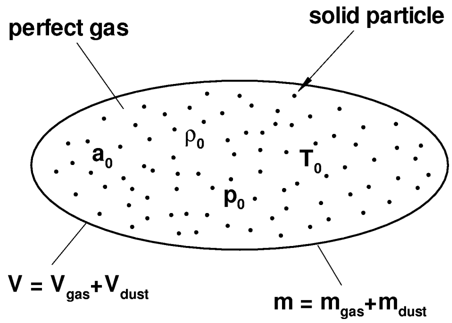

Fig. 2. Dusty gas in its reference state. The solid particles are continuously distributed in the perfect gas. The solid particles in their totality are referred to as dust. The volume V of the mixture consists of the volume of the perfect gas V_{gas} and the volume of the dust V_{dust}. The mass of the mixture m is constituted by the mass of the gas m_{gas} and the mass of the dust m_{dust}. a_0, p_0, T_0 and ρ_0 are the speed of sound, the pressure, the temperature and the density of the mixture, respectively, at the reference state.

specifies the mass fraction of the dust. $k_{\text{dust}} = 0$ represents the dust free case. Assuming a volumetric extension of the dust, the volume of the mixture V is constituted by the volume of the perfect gas V_{gas} and the volume of the dust V_{dust}:

$$V = V_{\text{gas}} + V_{\text{dust}}.$$

As the dust is regarded as an incompressible solid phase, V_{dust} is completely specified in terms of m_{dust}, i.e.,

$$V_{\text{dust}} = \frac{m_{\text{dust}}}{\rho_{\text{dust}}}, \quad \rho_{\text{dust}} = \text{const.} \tag{5}$$

The dust's volumetric extension is represented by the parameter G defined as

$$G := \frac{\rho_{\text{dust}}}{\rho_{\text{gas}}}, \tag{6}$$

where $\rho_{\text{gas}} = m_{\text{gas}}/V_{\text{gas}}$. Vanishing volumetric extension of the dust, i.e., $V_{\text{dust}} \to 0$, implies $G \to \infty$. Since according to equation (5) the dust's volume remains always constant, the perfect gas assumption does not apply to the dust phase.

The volumetric fraction of the dust in the mixture at a considered state (ρ, T) is simply given by

$$Z = \frac{V_{\text{dust}}}{V} = Z_0 \frac{\rho}{\rho_0}, \tag{7}$$

where Z_0 represents the corresponding value at the reference state. Z_0 is completely determined by the dust parameters k_{dust} and G, respectively, as follows

$$Z_0 = \frac{V_{\text{dust}}}{V} = \frac{k_{\text{dust}}}{(1 - k_{\text{dust}})G + k_{\text{dust}}}. \tag{8}$$

Representing the solid phase behaviour, Z_0 accounts for the loss of compressibility of the dusty gas. The particular case $G = 1$ is associated with $Z_0 = k_{\text{dust}}$. The dust and the perfect gas are then virtually undiscernable concerning their specific volume, but the dust's volume fraction still behaves as an incompressible solid phase.

Summing up, the dusty gas should be seen as a pure perfect gas which is contaminated by small particles and not as a mixture of two perfect gases. The dust particles are assumed to be highly dispersed in the gas phase such that the dusty gas can be considered as a continuous medium where the conservation equations (1)–(3) apply. All relaxation processes are excluded such that no relative motion and no temperature differences between gas and particles occure. In addition, the dust particles are assumed to have no thermal motion, and, hence, they do not contribute to the pressure of the mixture [5]. As a consequence, the pressure p and the temperature T of the entire mixture satisfy the thermal equation of state of the perfect gas partition. Referred to the mixture's unit mass, the thermal equation of state reads then

$$p = \frac{1 - k_{\text{dust}}}{1 - Z} R_{\text{gas}} \rho T, \tag{9}$$

where R_{gas} is the specific gas constant of the perfect gas.

The mixture's specific internal energy is constituted by the internal energy of the perfect gas and the internal energy of the dust. The caloric equation of state reads then as [5]

$$e = \frac{1 - Z}{\Gamma - 1}\left(\frac{p}{\rho}\right). \tag{10}$$

Therein

$$\Gamma = \frac{\gamma + \delta\beta}{1 + \delta\beta}, \tag{11}$$

where $\gamma = (c_p/c_v)_{\text{gas}}$ is the ratio of the specific heats of the perfect gas.

$$\delta = \frac{k_{\text{dust}}}{1 - k_{\text{dust}}} \quad \text{and} \quad \beta = \frac{(c)_{\text{dust}}}{(c_v)_{\text{gas}}}$$

are constant parameters, where $(c)_{\text{dust}}$ is the specific heat of the dust. The incompressibility assumption of the dust implies equality of the dust's volumetric and isobaric specific heats

$$(c_p)_{\text{dust}} = (c_v)_{\text{dust}} = (c)_{\text{dust}}. \tag{12}$$

From the two equations of state (9) and (10) the mixture's speed of sound [5] can be obtained as follows

$$a^2 = \frac{\Gamma(1 - k_{\text{dust}})}{(1 - Z)^2} R_{\text{gas}} T. \tag{13}$$

Fig. 3 displays the speed of sound at the reference state vs. k_{dust} at fixed G. Evidently a is increasing with k_{dust} for a small value of G, e.g., $G = 1$, and decreasing as G becomes large, e.g., $G \to \infty$. The reason for this behaviour is demonstrated in Figs. 4 and 5.

For $G = 1$, Eq. (8) gives $Z_0 = k_{\text{dust}}$, and Eq. (9) can be rewritten for the reference state, where $Z = Z_0$,

$$\rho_0 = \frac{p_0}{R_{\text{gas}} T_0},$$

and, as shown in Fig. 4, the mixture's density ρ_0 does not vary with k_{dust}. In this case the addition of dust does not increase the mixture's inertia, and does not additionally retard any wave propagation. On the other hand, the dust's volumetric fraction Z_0 linearly increases with k_{dust}, as the volumetric extension of the dust V_{dust} increases. Underlying Eqs. (5) and (12) the dust is considered as an incompressible solid phase. The occurrence of an incompressible phase in a mixture basically lowers the mixture's compressibility. This decrease of the mixture's compressibility is reflected by the dusty gas model through an increase of the mixture's speed of sound as k_{dust} increases in Eq. (13) for $G = 1$.

For $G \to \infty$ the dust's volumetric extension is vanishing ($V_{\text{dust}} = 0$). As can be seen in Fig. 5 the dust's volumetric fraction Z_0 is equal to zero according to Eq. (8). In this case the dust load is noticed only as an additional mass. Basically, the perfect gas just becomes heavier. This is reflected by the nonlinear increase of the density ρ_0 with k_{dust}, i.e.,

$$\rho_0 = \left(\frac{p_0}{R_{\text{gas}} T_0}\right)\left[\frac{1}{1 - k_{\text{dust}}}\right].$$

In this case the dust load only increases the dusty gas's inertia, while it does not affect its compressibility due to the zero volumetric extension of the dust. This feature is reflected by the decrease of the mixture's speed of sound with k_{dust} for large G as shown in Fig. 3.

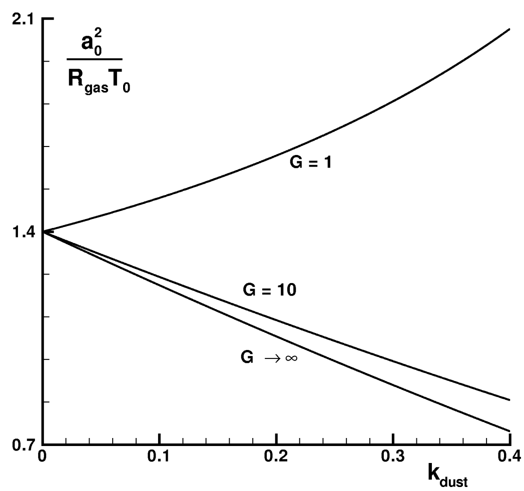

Fig. 3. The speed of sound $a_0^2/(R_{\text{gas}} T_0)$ of the mixture at the reference state vs. the mass fraction k_{dust} of the dust.

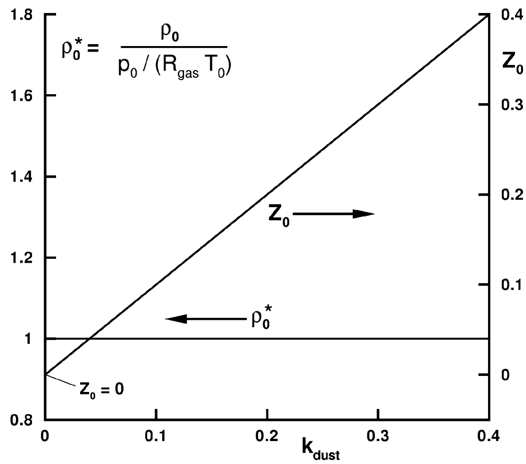

Fig. 4. Variation of the dust's volumetric fraction $Z_0 = V_{\text{dust}}/V$ and the mixture's nondimensionalized density $\rho_0^* = \rho_0/(p_0/R_{\text{gas}}T_0)$ with the dust's mass fraction k_{dust} obtained at the mixture's reference state for a volumetric parameter $G = 1$.

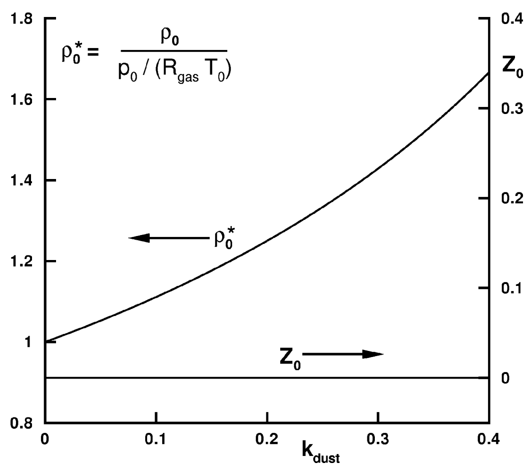

Fig. 5. Variation of the dust's volumetric fraction $Z_0 = V_{\text{dust}}/V$ and the mixture's nondimensionalized density $\rho_0^* = \rho_0/(p_0/R_{\text{gas}}T_0)$ with the dust's mass fraction k_{dust} obtained at the mixture's reference state for a volumetric parameter $G \to \infty$.

3. Self-similarity transformation

As shown in Fig. 1 the flow field is bounded by the piston and the shock, respectively. In the framework of self-similarity [8] the velocity $U_p = \mathrm{d}r_p/\mathrm{d}t$ of the piston is required to follow a power law which reads

$$U_p = \frac{\mathrm{d}r_p}{\mathrm{d}t} = U_0\left(\frac{t}{t_0}\right)^n, \tag{14}$$

where t_0 denotes a reference time. (14) encompasses three cases of the formation of a strong shock, where the counter-pressure ahead of the shock can be neglected: for $n = 0$ the piston instantaneously starts at $t = 0$ with the velocity U_0, which remains constant in time. A strong shock is formed from the beginning, if the piston velocity U_0 is sufficiently high. For $n > 0$ the piston is continuously accelerated. A shock is formed which reaches the strong shock limit at large times. For $-1 < n < 0$ the piston velocity jumps at $t = 0$ from zero to infinite velocity leading to the formation of a strong shock in the initial phase. Cylindrical and spherical geometry ($r > 0$) do not permit any exponent $n \leqslant -1$ for physical reasons. Concerning the shock boundary condition self-similarity requires that the velocity of the shock $U_s = \mathrm{d}r_s/\mathrm{d}t$ is proportional to the velocity of the piston:

$$U_s = \frac{\mathrm{d}r_s}{\mathrm{d}t} = CU_0\left(\frac{t}{t_0}\right)^n, \tag{15}$$

where C is a constant. The time and space co-ordinate can be transformed into a dimensionless self-similarity variable as follows

$$\lambda := \frac{r}{r_s} = \left[\frac{(n+1)t_0^n}{U_0 C}\right]\left(\frac{r}{t^{n+1}}\right). \tag{16}$$

Evidently, $\lambda = \lambda_p = r_p/r_s$ at the piston and $\lambda = 1$ at the shock. Expressing the mixture's velocity u, its density ρ and its pressure p as

$$u = \Phi(\lambda)\left(\frac{r}{t}\right), \qquad \rho = \Lambda(\lambda)\rho_0, \qquad p = \Psi(\lambda)\left(\frac{r^2}{t^2}\right)\rho_0, \tag{17}$$

where Φ, Λ and Ψ are functions of λ only, the conservation equations (1)–(3) can be transformed into a system of ordinary differential equations with respect to λ. This system reads

$$\begin{bmatrix} \Lambda & [\Phi-(n+1)] & 0 \\ [\Phi-(n+1)] & 0 & \dfrac{1}{\Lambda} \\ \Gamma\Psi & 0 & [\Phi-(n+1)](1-Z) \end{bmatrix} \begin{pmatrix} \dfrac{\mathrm{d}\Phi}{\mathrm{d}\ln\lambda} \\ \dfrac{\mathrm{d}\Lambda}{\mathrm{d}\ln\lambda} \\ \dfrac{\mathrm{d}\Psi}{\mathrm{d}\ln\lambda} \end{pmatrix} = \begin{bmatrix} -\Lambda\Phi(j+1) \\ -\dfrac{2\Psi}{\Lambda} + \Phi - \Phi^2 \\ (2\Psi)(1-\Phi)(1-Z) - \Gamma\Psi\Phi(j+1) \end{bmatrix}. \tag{18}$$

Eq. (18) does not explicitly depend on the time. Self-similarity requires that the set of boundary conditions does not depend explicitly on the time either. Using Eqs. (15) and (17) in the strong shock limit, the boundary conditions at $\lambda = 1$ can be transformed into

$$\Phi(\lambda=1) = \left(\frac{2}{\Gamma+1}\right)(1-Z_0)(n+1), \tag{19}$$

$$\Lambda(\lambda=1) = \left(\frac{\Gamma+1}{\Gamma-1+2Z_0}\right), \tag{20}$$

$$\Psi(\lambda=1) = \left(\frac{2}{\Gamma+1}\right)(1-Z_0)(n+1)^2. \tag{21}$$

The piston's path coincides at $\lambda_p = r_p/r_s$ with a particle path. Using Eqs. (14) and (17) the relation

$$\Phi(\lambda=\lambda_p) = (n+1) \tag{22}$$

can be derived. Since the value of λ_p at the piston is not known prior to the integration, the system equation (18) is integrated replacing $\ln\lambda$ by Φ as the independent variable, such that $\lambda = \lambda(\Phi)$.

4. Results

Eq. (18) was integrated using a fourth-order Runge–Kutta algorithm yielding the variables Φ, Ψ, Λ as functions of λ. The present results were obtained assuming spherical geometry, where $j = 2$. The isentropic exponent and the dust model parameter β occuring both in (11) were set to $\gamma = 1.4$ and $\beta = 1$ [6], respectively. The mass fraction k_{dust} was varied between the dust free case $k_{\text{dust}} = 0$ and $k_{\text{dust}} = 0.4$. The volumetric parameter G occuring in (8) and (7) was set to $G = 1$, $G = 10$ and $G \to \infty$. It is noted that the present parametric study neglects deviations of the employed dust model [5] which have surely to be exspected for larger volume or mass fractions.

Using the Eqs. (19)–(21) the individual flow variables u, p and ρ can be related to their corresponding values immediately behind the shock, which are denoted by the subscript s, as follows

$$\frac{u}{u_s} = \frac{\Phi}{\Phi(\lambda=1)}\lambda, \qquad \frac{\rho}{\rho_s} = \frac{\Lambda}{\Lambda(\lambda=1)}, \qquad \frac{p}{p_s} = \frac{\Psi}{\Psi(\lambda=1)}\lambda^2.$$

Figs. 6–8 display the flow variables related to their values at the shock obtained for the cases $n = -0.2$, $n = 0$ and $n = 2$, for a volumetric parameter $G = 10$ and a dust's mass fraction $k_{\text{dust}} = 0.2$. As can be seen from Eq. (18), there is a singularity at the piston, where $\Phi = n+1$, because the equation for Λ becomes singular there. In the constant piston velocity case associated with $n=0$ this singularity is regular, and a finite solution for Λ is obtained as shown in Fig. 7. In the case of the accelerated ($n=2$) and the decelerated $n=-0.2$ piston, the singularity is irregular, and the derivative of the density tends to positive and negative infinity, respectively. As is shown in Figs. 8 and 6, the density at the piston approaches infinity for $n=2$ and approaches zero for $n=-0.2$, respectively. This singulartity can be physically interpreted as follows: the path of the accelerated piston converges with the path of the particle immediately ahead condensing the gas to infinity whereas the path of the decelerated piston diverges from the path of the particle immediately ahead rarifying the gas.

Basically, the pressure and the velocity radially decrease due to the spherical geometry, except for the decelerated piston, where the pressure radially increases (Fig. 6). In the latter the decrease in pressure due to geometry is not compensated by the compression of the piston. Within the framework of the present dust model, the results described so far are valid for any dust load including the dust-free case.

The addition of the dust can have a major impact on the velocity ratio shock to piston. Recalling Eqs. (15) and (16) the inverse of the similarity variable can be rewritten at the piston as

$$\frac{1}{\lambda_p} = \frac{U_s}{U_p} = C.$$

Fig. 9 displays the velocity ratio $C = U_s/U_p$ varying with the dust's mass fraction k_{dust} for different volumetric parameters G obtained in the accelerated piston case, $n=2$. For $G=1$ the velocity of the shock U_s noticeably increases with k_{dust}, while it slightly decreases with k_{dust} for large values like $G \to \infty$. Basically, this behaviour mirrors the two counterplaying effects

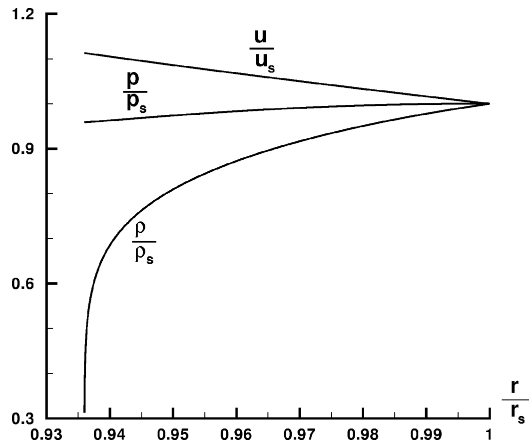

Fig. 6. The radial profiles of the velocity u, the pressure p and the density ρ for an decelerated piston ($n=-0.2$). All quantities are related to their corresponding values u_s, p_s, ρ_s and r_s at the shock.

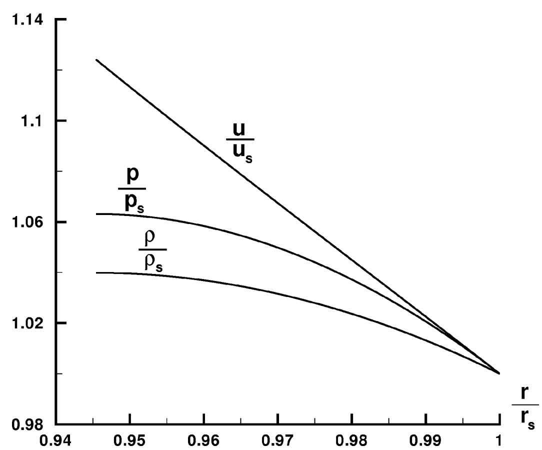

Fig. 7. The radial profiles of the velocity u, the pressure p and the density ρ for a constant velocity piston ($n=0$). All quantities are related to their corresponding values u_s, p_s, ρ_s and r_s at the shock.

accounted for by the dusty gas model, i.e. the dusty gas's lowered compressibility on the one hand and the dust load's additional inertia on the other hand. Analogously to the speed of sound, as discussed in Fig. 3, there is also an increase/decrease of the shock velocity depending on whether the dust load's incompressibility or inertia effect prevails. The decelerating effect on the propagation of the strong shock due to the dust load's inertia is markedly weaker though. The accelerating/decelerating effect of the dust applies to all three basic cases of the piston's motion as shown for $G = 1$ and $G \to \infty$ in Table 1.

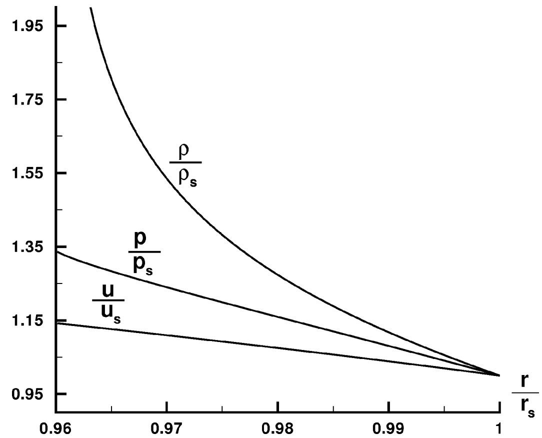

Fig. 8. The radial profiles of the velocity u, the pressure p and the density ρ for an accelerated piston ($n = 2$). All quantities are related to their corresponding values u_s, p_s, ρ_s and r_s at the shock.

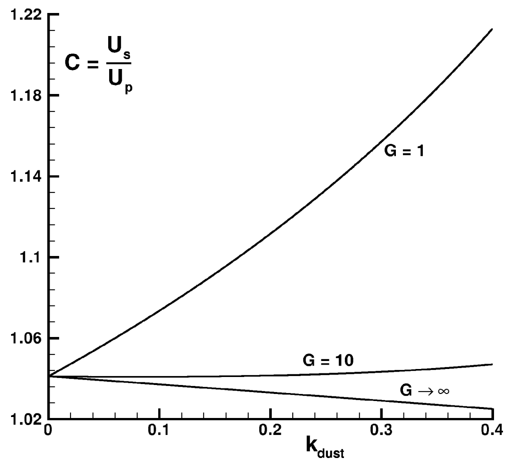

Fig. 9. The shock-to-piston velocity ratio $C = U_s/U_p$ vs. the mass fraction k_{dust} of the dust.

Table 1
The ratio $C = U_s/U_p$ with U_s being the shock velocity and U_p being the piston velocity obtained for the volumetric parameters $G = 1$ and $G \to \infty$ and for an increasing mass fraction of the dust k_{dust}. $n = -0.2, 0, 2$ refer to a decelerated, constant velocity and accelerated piston, respectively

		$G = 1$			$G \to \infty$	
k_{dust}	$n = -0.2$	$n = 0$	$n = 2$	$n = -0.2$	$n = 0$	$n = 2$
0.0	1.07318	1.06053	1.04112	1.07318	1.06053	1.04112
0.1	1.10335	1.09155	1.07354	1.06663	1.05495	1.03713
0.2	1.13901	1.12812	1.11161	1.05994	1.04927	1.03312
0.3	1.18198	1.17207	1.15665	1.05310	1.04350	1.02907
0.4	1.23503	1.22618	1.21294	1.04610	1.03763	1.02500

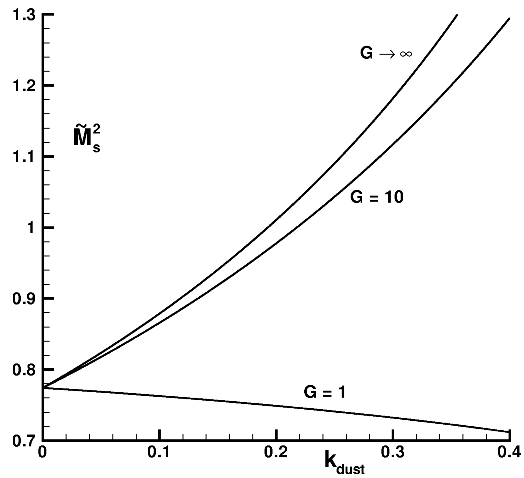

Fig. 10. Variation of the modified square of the shock Mach number $\widetilde{M}_s^2 = A(U_s^2/a_0^2)$, where A is the constant factor $A = (R_{\mathrm{gas}}T_0)/U_p^2$, with the mass fraction k_{dust} of the dust.

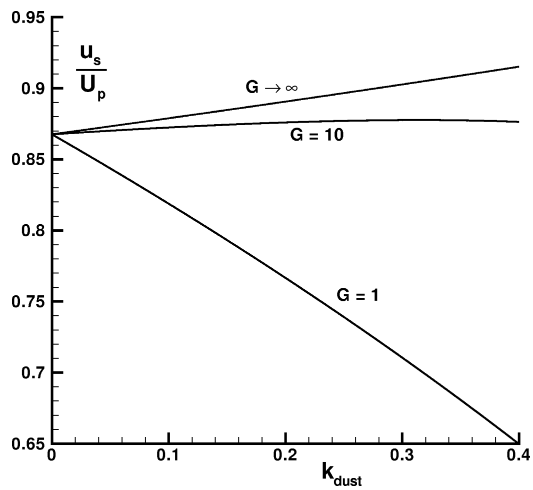

Fig. 11. Variation of the mixture's particle velocity immediately behind the shock u_s related to the piston velocity U_p with the mass fraction of the dust k_{dust}.

A higher shock velocity does not automatically lead to a stronger shock. It rather depends on the square of the shock Mach number

$$M_s^2 = \frac{U_s^2}{a_0^2}.$$

As shown in Fig. 10 the increase in the shock velocity U_s^2 is evidently overcompensated by the increase of the speed of sound a_0 resulting in a decreasing square of the Mach number for $G = 1$, and the shock becomes weaker in this case. On the other hand, despite the lower shock speed, the case $G \to \infty$ exhibits a higher square of the Mach number as the speed of sound a_0 decreases with k_{dust}. This dependence on the square of the Mach number becomes obvious in Fig. 11 showing the mixture's particle velocity immediately behind the shock referred to the velocity of the piston u_s/U_p vs. k_{dust}. Contrary to the relative shock velocity U_s/U_p, the relative particle velocity u_s/U_p is decreasing in the case $G = 1$, because the shock becomes weaker. It is increasing in the case $G \to \infty$, where the shock becomes stronger.

5. Conclusion

The present work investigates the self-similar flow field between a piston and a shock in a dusty gas. For the employed dust model it is shown that the flow field is mainly affected by the dust's impact on the speed of sound. The addition of dust affects the speed of sound basically in two ways: Firstly, it may increase the inertia; secondly, it may decrease the compressbility of the mixture. The speed of sound is decreased by the first, and it is increased by the latter. Depending on whether the mixture becomes 'heavier' or 'less compressible', a lower/higher shock speed is obtained. It is further demonstrated that the speed of sound relatively decreases/increases faster than the shock speed as the dust load becomes higher, leading to a stronger/weaker shock.

Appendix: Derivation of the jump conditions at a strong shock (Eqs. (19)–(21))

Neglecting the counter-pressure the jump conditions at a shock front read

$$\rho_0 U_s = \rho_s (U_s - u_s), \tag{A.1}$$

$$\rho_0 U_s^2 = p_s + \rho_s (U_s - u_s)^2, \tag{A.2}$$

$$\frac{U_s^2}{2} = e_s + \frac{p_s}{\rho_s} + \frac{(U_s - u_s)^2}{2}. \tag{A.3}$$

(A.1) and (A.2) can be rewritten as

$$\rho_s = \rho_0 \frac{1}{1 - \frac{u_s}{U_s}}, \tag{A.4}$$

$$\frac{p_s}{\rho_0 U_s^2} = \frac{u_s}{U_s}. \tag{A.5}$$

Introducing (10), (A.4) and (A.5) into (A.3) yields after some manipulation

$$\frac{u_s}{U_s} = \left(\frac{2}{\Gamma + 1}\right)(1 - Z_0). \tag{A.6}$$

Using the self-similarity transforms (17) and Eq. (15) Eqs. (A.4)–(A.6) can be finally rewritten as (19)–(21)

$$\Phi(\lambda = 1) = \left(\frac{2}{\Gamma + 1}\right)(1 - Z_0)(n + 1),$$

$$\Lambda(\lambda = 1) = \left(\frac{\Gamma + 1}{\Gamma - 1 + 2Z_0}\right),$$

$$\Psi(\lambda = 1) = \left(\frac{2}{\Gamma + 1}\right)(1 - Z_0)(n + 1)^2.$$

References

[1] T. Suzuki, F. Higashino, A. Takano, The propagation of reacting blast waves through inert particle clouds, Acta Astronaut. 3 (1976) 517–529.
[2] F. Higashino, T. Suzuki, The effect of particles on blast wave in a dusty gas, Z. Naturforsch. 35a (1980) 1330–1336.
[3] S.I. Pai, S. Menon, Z.Q. Fan, Similarity solution of a strong shock wave propagation in a mixture of gas and dusty particles, Int. J. Engng. Sci. 18 (1980) 1365–1373.
[4] G. Narasimhulu Naidu, K. Vekatanandam, M.P. Ranga Rao, Approximate analytical solutions for self-similar flows of a duty gas with variable energy, Int. J. Engng. Sci. 23 (1985) 39–49.
[5] S.I. Pai, Two-phase Flows, Vieweg Verlag, 1977.
[6] R. Miura, I.I. Glass, Development of the flow induced by a piston moving impulsively in a dusty gas, Proc. Roy. Soc. London A 3 (1985) 295–309.
[7] J.P. Vishwakarma, Propagation of shock waves in a dusty gas with exponentially varying density, Eur. Phys. J. B 16 (2000) 369–372.
[8] L.I. Sedov, Similarity and Dimensional Methods in Mechanics, Academic Press, 1959.